Math 253 Overview Ch.16

Chapter 16 – Integration

16.1 Definite integral

Area and signed-volume interpretations of integrals

16.2 Iterated integrals (in Cartesian coordinates)

Outermost limits of double integral are constants

Setting up double integrals over non-rectangular regions

Deciding on an order of integration: dy dx versus dx dy

Reversing the order of integration (draw a picture!)

Some applications of double integrals:

- Find Mass of a 2D plate, given density function.
- Find Area of a 2D region.
- Find the (signed) volume between a surface and the xy-plane.
- Find the volume between an upper surface and a lower surface.
- Find the average value of a function over a 2D region.
- 16.4 Polar coordinates (Double integrals revisited)
  - Given a polar integral, be able to sketch the 2D region of integration.
  - Be able to convert Cartesian integrals to polar integrals (draw a picture!)
  - Don't forget to convert the formula you are integrating into a polar function.
  - x = r cos(theta),   y = r sin(theta),   (x^2+y^2)^(1/2) = r
  - dA = r dr dtheta (or r dtheta dr)
  - Applications: Same as applications of double-integrals in 16.2

16.3 Triple integrals (Cartesian coordinates)

Outermost limits of triple integral are constants.

Six possible orders of integration.

The limits of the outer pair of integrals can be viewed as a double-integral covering the projection of the 3D surface into the coordinate plane corresponding to the last two differentials. For example, if the order is dx dy dz, then the outer two-integrals may be viewed as a double-integral covering the 2D projection of the shape into the yz-plane.

Setting up triple integrals over non-cubic solids.

Given a triple integral, you should be able to sketch the 3D region of integration.

Some applications of triple-integrals:

- Find the Mass of a 3D solid, given the density function.

- Find the Volume of a 3D solid (note: we performed this calculation somewhat differently in 16.2)
- Find the average value of a function over a 3D region.

16.5 Cylindrical and Spherical coordinates (Triple integrals revisited)

Given a triple integral in either cylindrical or spherical coordinates, draw the 3D region of integration.

Be able to convert cartesian triple integrals to cylindrical or spherical coordinates (Hints: draw a picture! and don't forget to convert the function that you are integrating to the new coordinate system.)

Know the conversions from Cartesian to each alternate coordinate systems.

Applications: Same as applications of triple-integrals in 16.3

Cylindrical coordinates

- dV = r dz dr dtheta (or any of 6 other arrangements)
  x=?, y=?, z=?, (x^2+y^2)^(1/2) = ?

Spherical coordinates

- dV = rho^2 sin(phi) drho dphi dtheta
  x=?, y=?, z=?, (x^2+y^2+z^2)^(1/2) = ?